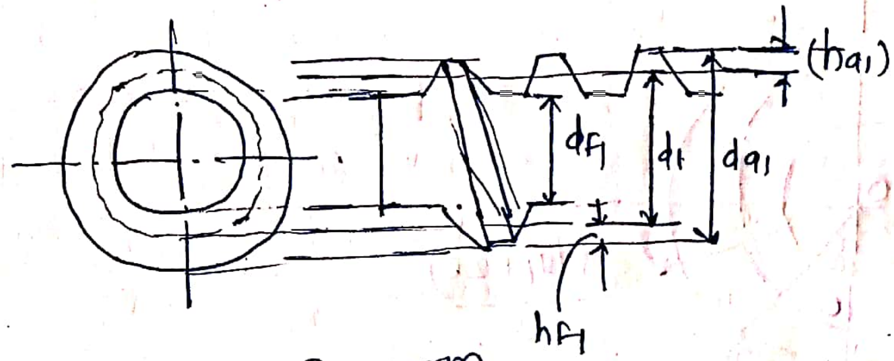
Proportions of Worm Gears :-



Dimensions for worm

$h_{a1} = m$ = addendum

$h_{f1} = (2.2\cos\gamma - 1)\, m$ = dedendum

$c = 0.2 m \cos\gamma$ = clearance.

→ d_{a1} = outside dia. of worm

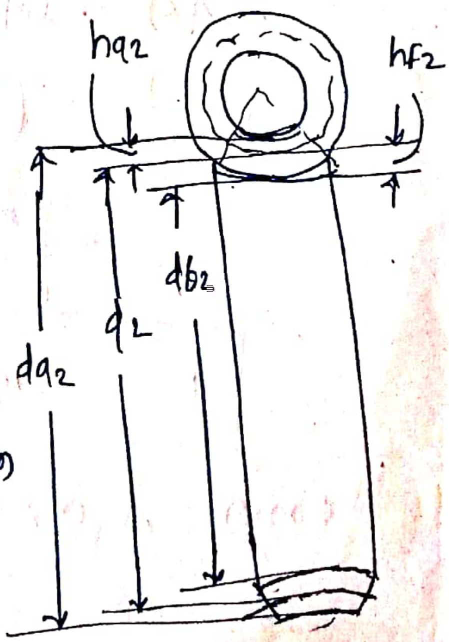
$= d_1 + 2h_{a1} = qm + 2m$

$d_{a1} = m(q+2)$

→ d_{f1} – root diameter of worm

$= d_1 - 2f_1 = qm - 2m(2.2\cos\gamma - 1)$

$d_{f1} = m(q + 2 - 4.4\cos\gamma)$



Dimensions for worm wheel :-

$h_{a2} = m(2\cos\gamma - 1)$ = Addendum at throat

$h_{f2} = m(1 + 0.2\cos\gamma)$

= dedendum at median plane.

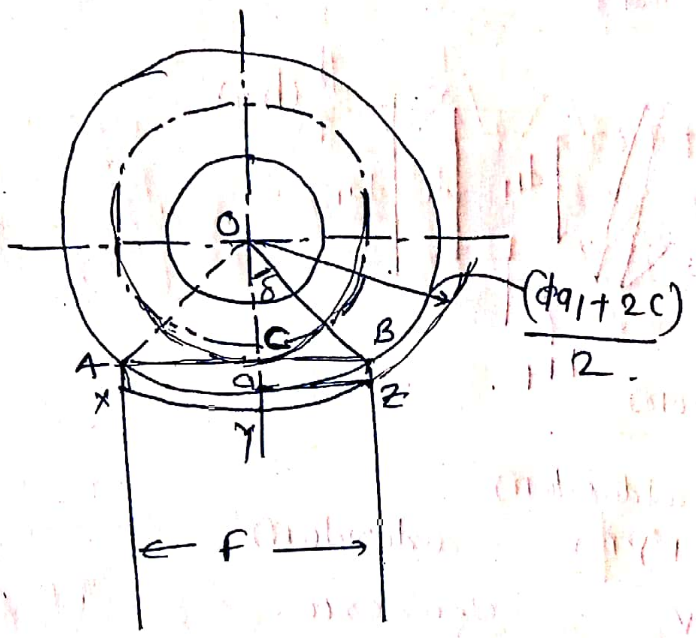
→ $d_{a2} = d_2 + 2h_{a2}$ = throat dia. of worm wheel

$= mz_2 + 2m(2\cos\gamma - 1)$

$= m(z_2 + 4\cos\gamma - 2)$

– $d_{f2} = d_2 + 2h_{f2} = mz_2 - 2m(1 + 0.2\cos\gamma)$

$= m(z_2 - 2 - 0.4\cos\gamma)$ = root dia. of worm wheel



Face width of worm wheel

~~Fro~~ face width (Effective) of worm wheel - by drawing tangent AB to PCD of worm.

- A,B are pts. of intersection of tangent & outside dia. of worm

From ΔAOC

$$(\overline{AC})^2 = (\overline{AO})^2 - (\overline{OC})^2$$

$$\left(\frac{F}{2}\right)^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2$$

$$= \left[\frac{m(q+2)}{2}\right]^2 - \left[\frac{qm}{2}\right]^2$$

$$\boxed{F = 2m\sqrt{q+1}}$$

From ΔOZC

$$\sin\delta = \frac{CZ}{OZ} = \frac{F/2}{[d_{a1}+2c/2]} \qquad \delta = \sin^{-1}\left(\frac{F}{d_{a1}+2c}\right)$$

→ Root length of worm wheel teeth is arc XYZ (l_r)

$$l_r = \text{Arc}(XYZ) = \frac{2\delta}{2\pi}\left[\pi(d_{a1}+2c)\right] = (d_{a1}+2c)\delta$$

$$l_r = (d_{a1}+2c)\sin^{-1}\left[\frac{F}{d_{a1}+2c}\right]$$

Force Analysis :-

$$(P_1)_t = \frac{2M_t}{d_1}$$

$$(P_1)_a = (P_1)_t \times \frac{(\cos\alpha \cos\gamma - \mu \sin\gamma)}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$(P_1)_r = (P_1)_t \times \frac{\sin\alpha}{(\cos\alpha \sin\gamma + \mu\cos\gamma)}$$

Example :-

A pair of worm and worm wheel is designated as 3/60/10/6. The worm is transmitting 5 kW power at 1440 rpm to the worm wheel. The coefficient of friction is 0.1 and the normal pressure angle is 20°. Determine the components of the gear tooth force acting on the worm & worm wheel. Draw FBD.

— $P_{kW} = 5$, $n = 1440$ rpm, $\mu = 0.1$, $\alpha = 20°$,

$z_1 = 3$, $z_2 = 60$, $q = 10$, $m = 6$ mm

$d_1 = qm = 60$ mm

$\tan\gamma = (z_1/q)$ $\quad \gamma = 16.7°$

$$M_t = \frac{60\times10^6\,(P_{kW})}{2\pi n_1} = 33157.28 \text{ Nmm}$$

$$(P_1)_t = \frac{2M_t}{d_1} = 1105.24 \text{ N}$$

$(P_1)_a = 2632.55$ N

$(P_1)_r = 1033.35$ N.

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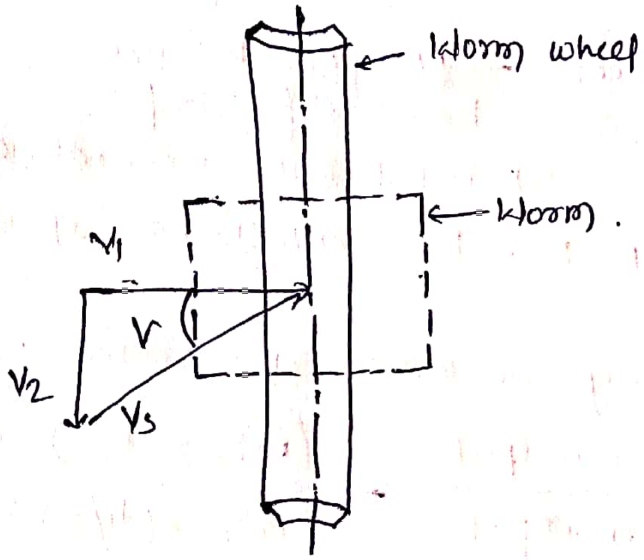
* Friction in worm Gear & Efficiency:-



- Coefficient of friction in worm wheel depends upon rubbing velocity (relative velocity betn worm & worm wheel)

V_1 = pitch line velocity of worm (m/s)

V_2 = ———— " ———— worm wheel (m/s)

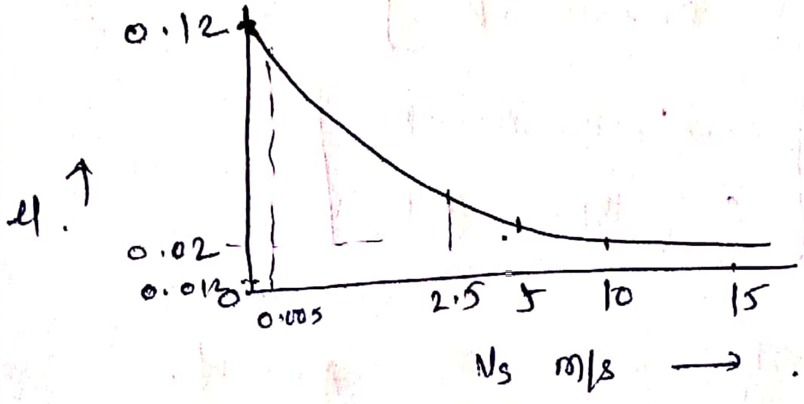
V_s = rubbing velocity m/s

$$V_1 = \frac{\pi d_1 n_1}{60 \times 10^3}$$

From velocity triangle $\cos r = \frac{V_1}{V_s}$

$$V_s = \frac{\pi d_1 n_1}{60 \times 10^3 \times \cos r}.$$

- Variation of coeff. of friction with rubbing velocity is as follows



- The values of coefficient of friction in above figure is based on assumptions as follows

i) worm wheel – phosphor bronze
worm – case hardened steel

ii) Gears are lubricated with mineral oil having viscosity of 16 – 130 centistrokes at 60°.

- The efficiency of worm gear drive is given by

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{(P_2)_t \times (d_2/2)\, n_2}{(P_1)_t \times (d_1/2) \times n_1}$$

As $\frac{n_2}{n_1} = \frac{1}{i}$

$$\frac{d_2/2}{d_1/2} = \frac{d_2}{d_1} = \frac{m z_2}{m q} = \frac{z_2}{q} = \frac{z_2/z_1}{q/z_1} = \frac{i}{(1/\tan\gamma)} = i \tan\gamma .$$

$$\eta = \frac{(P_2)_t}{(P_1)_t} \cdot i \tan\gamma \times \frac{1}{i} = \frac{(P_2)_t}{(P_1)_t} \tan\gamma$$

As we know $(P_2)_t = (P_1)_a = (P_1)_t \left[\frac{\cos\alpha \cos\gamma - \mu \sin\gamma}{\cos\alpha \sin\gamma + \mu \cos\gamma} \right]$

~~$\frac{}{}(P_1)_t =$~~

$$\eta = \tan\gamma \left[\frac{\cos\alpha \cos\gamma - \mu \sin\gamma}{\cos\alpha \sin\gamma + \mu \cos\gamma} \right]$$

$$= \frac{(1/\cos\gamma)}{(1/\sin\gamma)} \left[\frac{\cos\alpha \cos\gamma - \mu \sin\gamma}{\cos\alpha \sin\gamma + \mu \cos\gamma} \right]$$

$$\boxed{\eta = \frac{\cos\alpha - \mu \tan\gamma}{\cos\alpha + \mu \cot\gamma}}$$

Forward 50 %.

<u>Self locking</u>

$\mu > \tan \gamma$

or friction angle (ϕ) > lead angle (γ).

ples :-

① 1 kW power at 720 rpm is supplied to the worm shaft. The no. of starts for threads of worm is four with a 50 mm pitch circle diameter. The worm wheel has 30 teeth with a 5 mm module. The normal pressure angle is 20°. Calculate the efficiency of the worm gear drive and the power lost in friction.

→ $l = \pi m z_1 = 20\pi$ mm

$\tan \gamma = \frac{l}{\pi d_1} = 0.4$ $\quad \gamma = 21.8$

$V_s = \frac{\pi d_1 n_1}{60 \times 10^3 \cos \gamma} = 2.03$ m/s

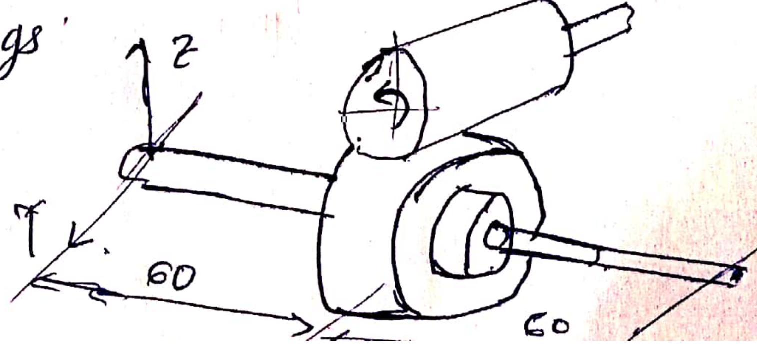
from Graph, coeff. of friction is 0.035.

$\eta = \frac{\cos \alpha - \mu \tan \gamma}{\cos \alpha + \mu \cot \gamma} = 90.12\ \%$.

Power lost in friction $= (1-\eta)\, P_{kW} = (1 - 0.9012)(1)$,

$= 0.0988$ kW $= 98.8$ kW.

② A 5 kW power at 720 rpm is supplied to the worm shaft as shown in fig. The worm gear drive is designated as 2/40/10/5. The worm has right hand threads and the pressure angle is 20°. The worm wheel is mounted between two bearings A & B. It can be assumed that bearing A is located at the origin of the coordinate system, and bearing B takes complete thrust load. Determine the reactions at the two bearings.



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→ $z_1 = 2$, $z_2 = 40$, $q = 10$, $m = 5$

$d_1 = qm = 50$ mm

$\tan \gamma = z_1/q = 0.2$ $\gamma = 11.31$

$$M_t = \frac{60 \times 10^6 (P \text{ in kW})}{2\pi n_1} = 66314.56 \text{ Nmm.}$$

$$(P_1)_t = \frac{2M_t}{d_1} = 2653 \text{ N}$$

$$V_s = \frac{\pi d_1 n_1}{60 \times 10^6 \cos\gamma} = 1.92 \text{ m/s.}$$

μ from graph $\mu = 0.035$

$$(P_1)_a = (P_1)_t \left(\frac{\cos\alpha \cos\gamma - \mu \sin\gamma}{\cos\alpha \cdot \sin\gamma + \mu \cos\gamma} \right)$$

$$= 11097 \text{ N.}$$

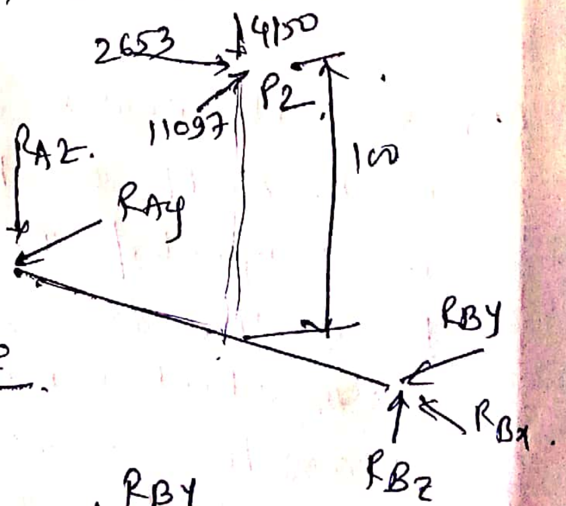
$$(P_1)_r = (P_1)_t \left(\frac{\sin\alpha}{\cos\alpha \sin\gamma + \mu \cos\gamma} \right) = 4150 \text{ N.}$$

– The force components acting on worm wheel are as follows

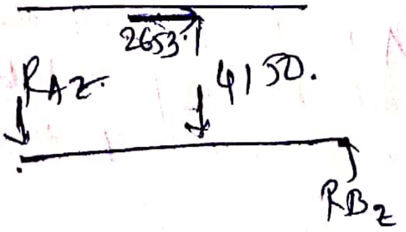
$(P_2)_t = (P_1)_a = 11097$ N

$(P_2)_a = (P_1)_t = 2653$ N

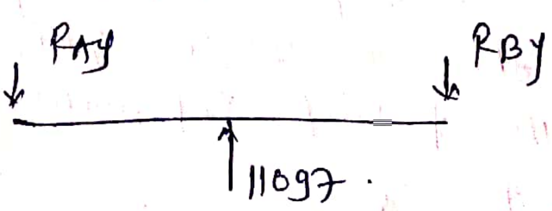
$(P_2)_r = (P_1)_r = 4150$ N.



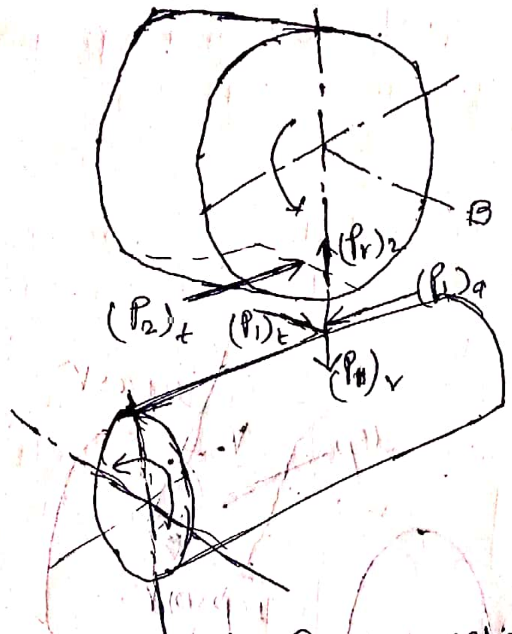
Vertical Plane

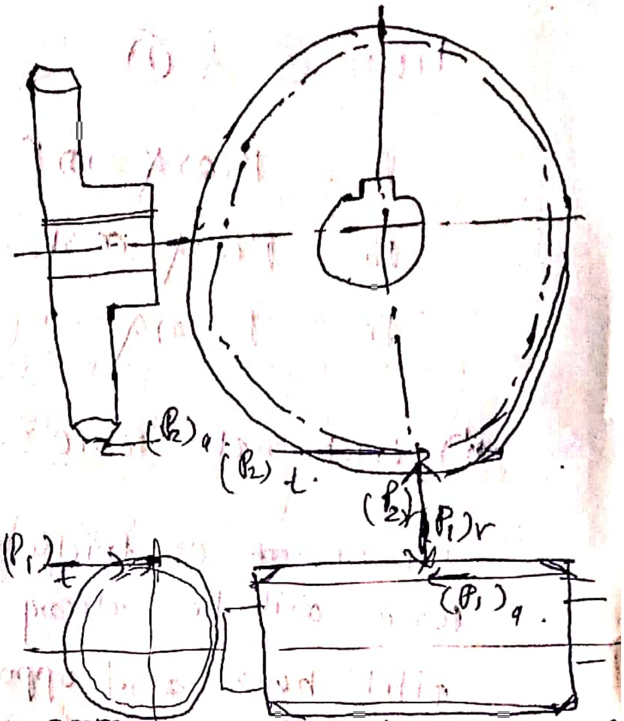


Horizontal Plane



* Force Analysis :-





* The resultant force acting on worm consist of two component i) component of normal reaction ii) frictional force.

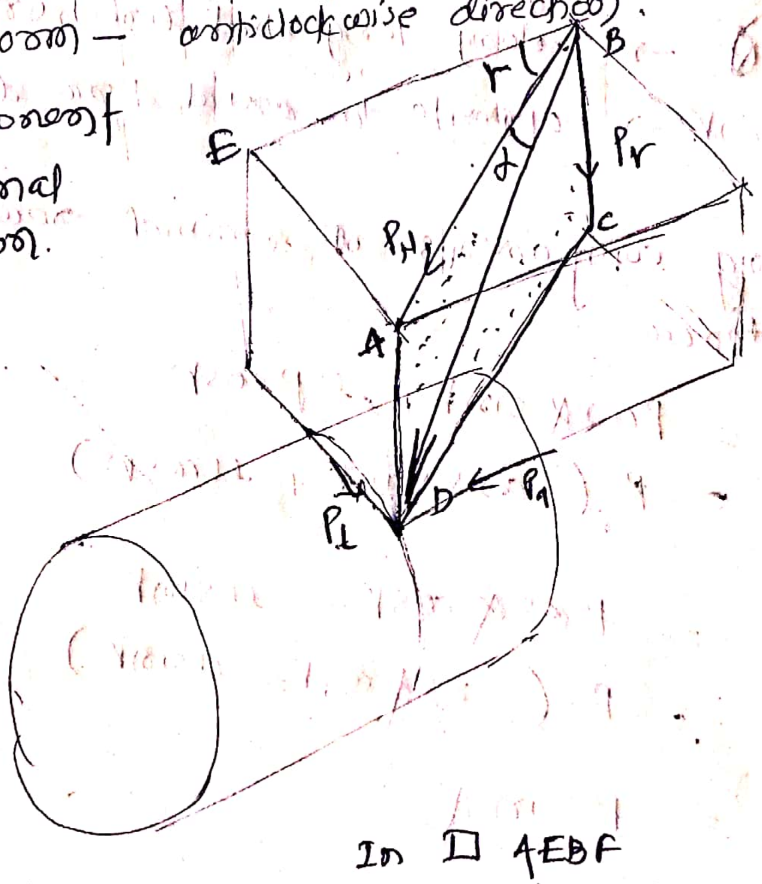
Assumptions :-

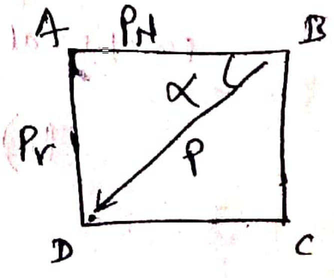
① worm – driving element, worm wheel – driven element

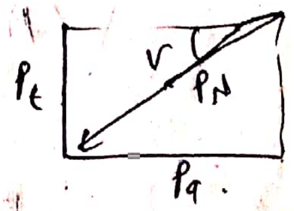
② worm – right handed threads

③ worm – anticlockwise direction.

I) Component of Normal Reaction.







In □ AEBF

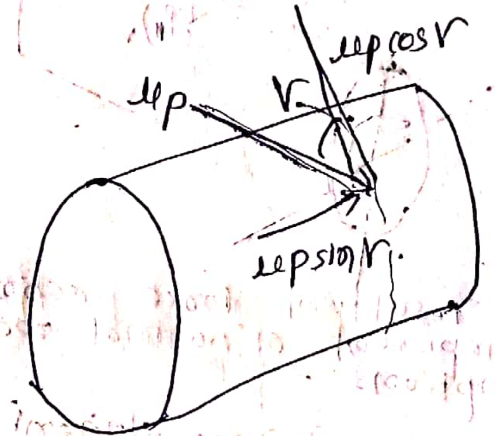
$$\left.\begin{aligned} P_N &= P\cos\alpha \\ P_r &= P\sin\alpha \end{aligned}\right\} ⓐ \qquad \left.\begin{aligned} P_a &= P_N\cos r \\ P_t &= P_N\sin r \end{aligned}\right\} ⓑ$$

From ⓐ & ⓑ

$$P_t = P\cos\alpha \sin\gamma$$
$$P_a = P\cos\alpha \cos\gamma$$
$$P_r = P\sin\alpha$$

ii) Frictional force :-

- Direction of frictional force will be along pitch helix and opposite to the direction of rotation.



- There are two components of frictional force
  i) $\mu P\cos\gamma \rightarrow$ along tangential direction
  ii) $\mu P\sin\gamma \rightarrow$ opposite to axial force direction.

Superimposing components of normal reaction & frictional force

$$(P_1)_t = P\cos\alpha\sin\gamma + \mu P\cos\gamma$$
$$= P(\cos\alpha\sin\gamma + \mu\cos\gamma)$$

$$(P_1)_a = P\cos\alpha\cos\gamma - \mu\sin\gamma$$
$$= P(\cos\alpha\cos\gamma - \mu\sin\gamma)$$

$$(P_1)_r = P\sin\alpha$$

In practice, tangential component $(P_1)_t$ on the worm is determined from torque that is transmitted from worm to worm wheel.

$$(P_1)_t = \frac{2M_t}{d_1}$$

$$(P_1)_a = (P_1)_t \frac{\cos\alpha \cos\gamma - \mu \sin\gamma}{\cos\alpha \sin\gamma + \mu \cos\gamma}$$

$$(P_1)_r = (P_1)_t \frac{\sin\alpha}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

* Strength rating of worm Gears :-

- Teeth of worm wheel are weaker than thread of worm.
- Design for strength can be based on lewis's eq^n as applied to worm wheel teeth.
- In this case, it is not necessary to design the worm on the basis of strength.
- worm gears are usually design according to national & international codes.
- The maximum permissible torque that the worm wheel can withstand without bending failure is given by lower of following two values.

$$(M_t)_1 = 17.65\, X_{b1} S_{b1}\, m\, l_r\, d_2 \cos\gamma$$

$$(M_t)_2 = 17.65\, X_{b2} S_{b2}\, m\, l_r\, d_2 \cos\gamma$$

$(M_t)_1, (M_t)_2$ = permissible torque on worm & worm wheel (N mm)

X_{b1}, X_{b2} = speed factor for strength of worm & worm wheel

S_{b1}, S_{b2} = bending stress factor of worm & worm wheel

m = module.

l_r = length of root of worm wheel teeth

$$l_r = (d_{a1} + 2c) \sin^{-1}\left(\frac{F}{(d_{a1} + 2c)}\right)$$

d_2 = PCD of worm wheel.

γ = Lead angle of worm.

- Power transmitting capacity based on beam strength is given by

$$kW = \frac{2\pi n M_t}{60 \times 10^6}$$

M_t is lower value bet^n $(M_t)_1$ & $(M_t)_2$

Value of bending stress factor S_b

Material	S_b
Phosphor bronze (centrifugally cast)	= 7
Phosphor bronze (sand cast & chilled)	= 6.4
Phosphor bronze (sand cast)	= 5
0.4 % carbon steel normalized (40C8)	= 14.1
0.55 % carbon steel normalised (55C8)	= 17.6
case hardened carbon steel (10C4, 14C6)	= 28.2
case hardened alloy steel (16Ni80Cr60 & 20Ni2Mo25)	= 33.11
Ni-Cr steel (13Ni3Cr80 & 15Ni4Cr1)	= 35.22

Example :-

A pair of worm & worm wheel is designated as 1/30/10/10. The input speed of the worm is 1200 rpm. The worm wheel is made of centrifugally cast, phosphor bronze and the worm is made of case-hardened carbon steel 14C6. Determine the power transmitting capacity based on the beam strength.

→ $n_1 = 1200$ rpm, $z_1 = 1$; $z_2 = 30$ teeth, $q = 10$, $m = 10$ mm.

$$i = \frac{z_2}{z_1} = 30$$

$$i = \frac{n_1}{n_2} \qquad n_2 = 40 \text{ rpm}$$

$$d_2 = m z_2 = 300 \text{ mm}$$

$$\tan \gamma = \frac{z_1}{q} = \frac{1}{10} \qquad \gamma = 5.71^\circ.$$

$$F = 2m\sqrt{q+1} = 2 \times 10 \sqrt{10+1} = 66.33 \text{ mm}$$

$$c = 0.2 m \cos\gamma = 0.2(10)\cos(5.71) = 1.99 \text{ mm}$$

$$d_{a1} = m(q+2) = 120 \text{ mm}$$

$$l_r = (d_{a1} + 2c) \sin^{-1}\left(\frac{F}{d_{a1}+2c}\right) = 69.998 \text{ mm}$$

For case-hardened carbon steel 14C6

$$S_{b1} = 28.2$$

for centrifugally cast phosphor bronze.

$$S_{b2} = 7$$

From graph

$X_{b1} = 0.25$ for $n_1 = 1200$ rpm

$X_{b2} = 0.48$ for $n_2 = 40$ rpm.

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$$(M_t)_1 = 17.65\, X_{b1}\, S_{b1}\, l_r\, m\, d_2 \cos\gamma$$

$$= 17.65\,(0.25)\,(28.2)\,(69.98)(10)\,(300)\,\cos(5.71)$$

$$= 2599671 \text{ Nmm}$$

$$(M_t)_2 = 17.65\, X_{b2}\, S_{b2}\, l_r\, m\, d_2 \cos\gamma$$

$$= 17.65\,(0.48)\,(7)\,(69.98)(10)(300)\,\cos(5.71)$$

$$= 1238992 2 \text{ Nmm}$$

- The lower value of torque on worm wheel is 1238992 2 Nmm.

- Power transmitting capacity based on beam strength.

$$P_{kW} = \frac{2\pi n_2 (M_t)}{60\times 10^6}$$

$$\boxed{P_{kW} = 51.9}$$

* Wear Rating of Worm Gear :-

The maximum permissible torque that the worm wheel can withstand without pitting failure is given by lower of two values

$$(M_t)_3 = 18.64 \cdot X_{c1} \, S_{c1} \, Y_z \, d_2^{1.8} \, m$$

$$(M_t)_4 = 18.64 \, X_{c2} \, S_{c2} \, Y_z \, d_2^{1.8} \, m$$

Where

$(M_t)_3$ & $(M_t)_4$ = permissible torque on worm wheel (Nmm)

X_{c1} , X_{c2} = speed factors for the wear of worm & worm wheel.

S_{c1} , S_{c2} = surface stress factors of the worm & worm wheel.

Y_z = zone factor.

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Example:-

A pair of worm & worm wheel is designated as 1/30/10/10. The input speed of the worm is 1200 rpm. The worm wheel is made of centrifugally cast, phosphor-bronze and worm is made of case-hardened carbon steel 14C6. Determine the power transmitting capacity based on the ~~beam~~ wear strength.

→ $n_1 = 1200$ rpm, $z_1 = 1$, $z_2 = 30$, $q = 10$, $m = 10$

$d_2 = 300$ mm $= m z_2$

for $q = 10$ & $z_1 = 1$

$Y_z = 1.143$ (from table)

For case hardened carbon steel 14C6

$S_{c1} = 4.93$

For centrifugally cast phosphor bronze

$S_{c2} = 1.55$

$$V_s = \frac{\pi d_1 n_1}{60 \times 10^3 \cos \gamma} = \frac{\pi \times (10 \times 10)(1200)}{60000 \times \cos(5.71)} = 6.315 \text{ m/s}$$

for $V_s = 6.315$ m/s & $n_1 = 1200$ rpm

$X_{c1} = 0.112$

for $V_s = 6.315$ m/s & $n_1 = 40$ rpm

$X_{c2} = 0.26$

$$(M_t)_3 = 18.64\, X_{c1}\, S_{c1}\, Y_z\, d_2^{1.8}\, m$$

$$= 18.64 \times 0.112 \times 4.93 \times 1.143 \times 300^{1.8} \times 10$$

$$= 3383570.4 \text{ Nmm}$$

$$(M_t)_4 = 18.64\, X_{c2}\, S_{c2}\, Y_z\, d_2^{1.8}\, m$$

$$= 18.64 \times 0.26 \times 1.55 \times 1.143 \times 300^{1.8} \times 10$$

$$= 2489535.8 \text{ Nmm}$$

The lower value of torque on worm wheel is 2469535.8 Nmm.

$$P_{kW} = \frac{2\pi n_2 (M_t)}{60 \times 10^6}$$

$$= 10.34$$

Practice Problem

Ex. (2) A pair of worm gears is designated as 1/40/10/4. The input speed of the worm shaft is 1000 rpm. The worm wheel is made of phosphor bronze (sand cast), while the worm of case hardened carbon steel 10C4. Determine power transmitting capacity based on wear strength.

Worm Gear :- thermal considerations :-

- The efficiency of a worm gear drive is low and work done by friction is converted into heat.
- When worm gears operate continuously, considerable amount of heat is generated.
- The rate of heat generated (H_g) is given by ——— ①

$$H_g = 1000\,(1-\eta)\ kW.$$

where H_g = rate of heat generated in 'W'

η = efficiency of worm gear

kW = power transmitted by gear (kW)

- The heat dev is dissipated through the lubricating oil to the housing wall and finally to the surrounding air.
- The rate of heat dissipated (H_d) by housing wall to surrounding air is given by ——— ②

$$H_d = k\,(t - t_o)\,A$$

where H_d = rate of heat dissipation (W)

k = overall heat transfer coeff. of housing wall ($W/m^2\,°C$)

t = temp. of lubricating oil

t_o = temp. of atm. air

A = effective surface area of housing

Equating ① & ②

$$1000\,(1-\eta)\,P_{kW} = k\,(t - t_o)\,A.$$

$$\boxed{P_{kW} = \frac{k\,(t - t_o)\,A}{100\,(1-\eta)}}$$

This eqn gives power transmitting capacity based on thermal considerations.

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This gives the resultant temp of the lubricating oil for given power transmitting capacity

$$t = t_o + \frac{1000(1-\eta)\,kW}{kA}$$

→ overall heat transfer coeff. under normal cooling condition with natural ~~convection~~ air circulation is 12 to 18 $W/m^2\,°C$.

→ for force convection 20-28 $W/m^2\,°C$

– temp of lubricating oil should not go above 95°C.

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Example :-

A worm gear box with an effective surface area of 1.5 m^2 is operating in still air with a heat transfer coeff. of 15 $W/m^2\,°C$. The temperature rise of the lubricating oil above the atmosphere temp. is limited to 50 °C. The worm gears are designated as 1/30/10/8. The worm shaft is rotating at 1440 rpm and the normal pressure angle is 20°. Calculate the power transmitting capacity based on thermal consideration.

→ $z_1 = 1,\quad z_2 = 30,\quad q = 10,\quad m = 8$ mm

$$\tan r = \frac{z_1}{q} = \frac{1}{10} = 0.1 \Rightarrow \boxed{r = 5.71°}$$

$$d = mq = 8(10) = 80 \text{ mm}$$

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos r} = \frac{\pi(80)(1440)}{60\,000 \cos(5.71)} = 6.06 \text{ m/s}$$

Based on $V_s = 6.06$ m/s, $\mu = 0.024$ → from table (design data book)

$$\eta = \frac{\cos\alpha - \mu\tan r}{\cos\alpha + \mu\cot r}$$

$$= \frac{\cos 20 - 0.024\,(0.1)}{\cos 20 + 0.024\,(1/0.1)}$$

$$\boxed{\eta = 0.7945}$$

$$P_{kW} = \frac{k(t - t_0)A}{1000(1-\eta)}$$

$$= \frac{15\,(50)(1.5)}{1000\,(1-0.7945)}$$

$$\boxed{P_{kW} = 5.47}$$

PRACTICE PROBLEM

Example 1. (2)

A pair of worm and worm wheel is designated as 1/30/10/10. The input speed of worm is 1200 rpm. The worm wheel is made of centrifugally cast phosphor bronze and worm is made of case hardened carbon steel 14C6. It has an effective surface area of $0.25\ m^2$. A fan is mounted on the worm shaft to circulate air over the surface of the fins. The coeff. of heat transfer can be taken as $25\ W/m^2\ {}^\circ C$. The permissible temp rise of the lubricating oil above the atmospheric temp. is $45^\circ C$. The coeff. of friction is 0.035 and normal pressure angle is 20°. Calculate the power transmitting capacity based on thermal considerations.